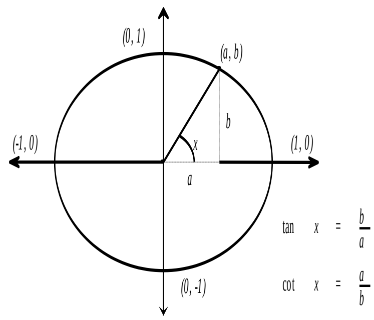
Graphing Tangent and Cotangent Functions

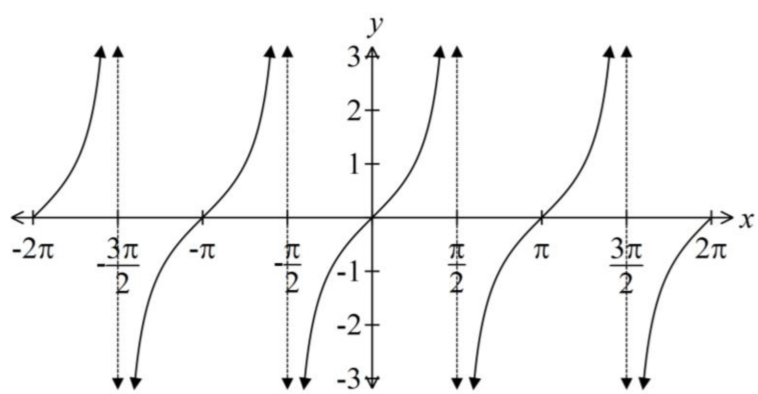
Let (a,b) be coordinates of points on the unit circle. For any given angle x, $\tan x = b/a$. This means that $y = \tan x$ is undefined whenever $a = 0$. For any given angle x, $\cot x = a/b$. This means that $y = \cot x$ is undefined whenever $b = 0$. Notice that it takes π radians for the values of the tangent and cotangent to make one complete cycle.



Graphing Tangent Functions:
The domain of $y = \tan x$ is the set of all real numbers except numbers of the form $\pi/2 + k\pi$, where k is an integer. The equations of the vertical asymptotes are $x = \pi/2 + k\pi$, where k is an integer.

Key points on the graph of $y = \tan x$:

x	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$
$y = \tan x$	undef.	-1	0	1	undef.



To graph $y = a\tan\left[b(x-c)\right] + d$:

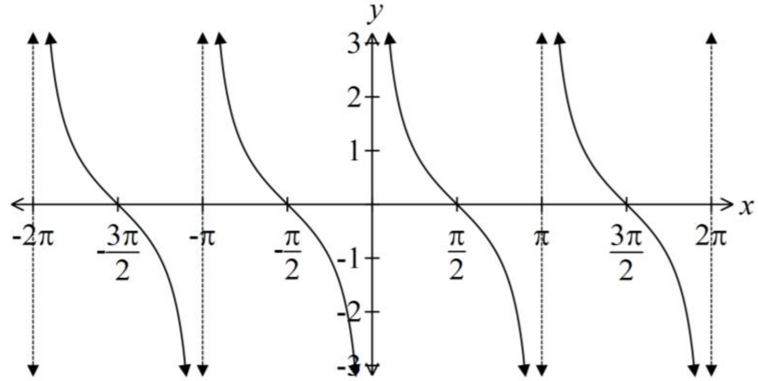
1. Start with the three key points on the graph of $y = \tan x$ and the equations of the asymptotes.
2. Find three key points and the asymptotes for $y = a\tan\left[b(x-c)\right] + d$ by:
   a. dividing each x-coordinate by b and adding c. (Treat the equations of the asymptotes like x-coordinates.)
   b. multiplying each y-coordinate by a and adding d.
3. Sketch one cycle of $y = a\tan\left[b(x-c)\right] + d$ through the three new points and approaching the new asymptotes.

★ **The period of $y = a\tan\left[b(x-c)\right] + d$ and $y = a\cot\left[b(x-c)\right] + d$ is π/b rather than $2\pi/b$.**

Graphing Cotangent Functions:
The domain of $y = \cot x$ is the set of all real numbers except numbers of the form $k\pi$, where k is an integer. The equations of the vertical asymptotes are $x = k\pi$, where k is an integer.

Key points on the graph of $y = \cot x$:

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$y = \cot x$	undef.	1	0	-1	undef.



Theorem 3.4. Properties of the Tangent and Cotangent Functions:

- The function $J(x) = \tan(x)$
  - has domain $\left\{x : x \neq \frac{\pi}{2} + \pi k,\ k \text{ is any integer}\right\}$
  - has range $(-\infty, \infty)$
  - is continuous and smooth on its domain
  - is odd
  - has period π
- The function $K(x) = \cot(x)$
  - has domain $\{x : x \neq \pi k,\ k \text{ is any integer}\}$
  - has range $(-\infty, \infty)$
  - is continuous and smooth on its domain
  - is odd
  - has period π

To graph $\boldsymbol{y = a\cot\left[b(x-c)\right]+d}$:

1. Start with the three key points on the graph of $y = \cot x$ and the equations of the asymptotes.
2. Find three key points and the asymptotes for $y = a\cot\left[b(x-c)\right]+d$ by:
   a. dividing each *x*-coordinate by *b* and adding *c*. (Treat the equations of the asymptotes like *x*-coordinates.)
   b. multiplying each *y*-coordinate by *a* and adding *d*.
3. Sketch one cycle of $y = a\cot\left[b(x-c)\right]+d$ through the three new points and approaching the new asymptotes.

Examples: Graph the following functions. Find the period and the equations of the asymptotes of each.

$y = \tan\left(\frac{1}{2}x\right)$

$y = \frac{1}{2}\cot\left(x+\frac{\pi}{3}\right)$

$y = 3\tan\left(2x+\frac{\pi}{2}\right)+1$

$y = 2\cot\left[3\left(x-\frac{\pi}{6}\right)\right]-1$